$$f(x) = (x-2)\sqrt{5-x}$$

① $\mathbb{D}\mathbb{B} = \{x;\ x \in \mathbb{R},\ x \leqq 5\}$; Symm. keine; $P_Y(0 \mid -2\sqrt{5})$ $\approx -4{,}5$

② Nst: $0 = (x-2)\sqrt{5-x}$

$\underline{\underline{x_{0_1} = 2 \qquad x_{0_2} = 5}}$

③ Unstet. keine

④ lok. Extrema: $f'(x) = \dfrac{-3x+12}{2\sqrt{5-x}}$

n.B: $0 = -3x + 12 \;\Rightarrow\; \underline{\underline{x_E = 4}}$

$f''(x) = \dfrac{3x-18}{4\sqrt{(5-x)^3}}$

h.B: $f''(4) = \dfrac{-6}{4} < 0 \;\Rightarrow\; \underline{\underline{P_H(4 \mid 2)}}$

⑤ WP: $f''(x) = \dfrac{3x-18}{4\cdot\sqrt{(5-x)^3}}$

n.B: $0 = 3x - 18 \;\Rightarrow\; x_W = 6 \notin \mathbb{D}\mathbb{B}$!

$\Rightarrow$ $\underline{\underline{\text{kein } P_W}}$

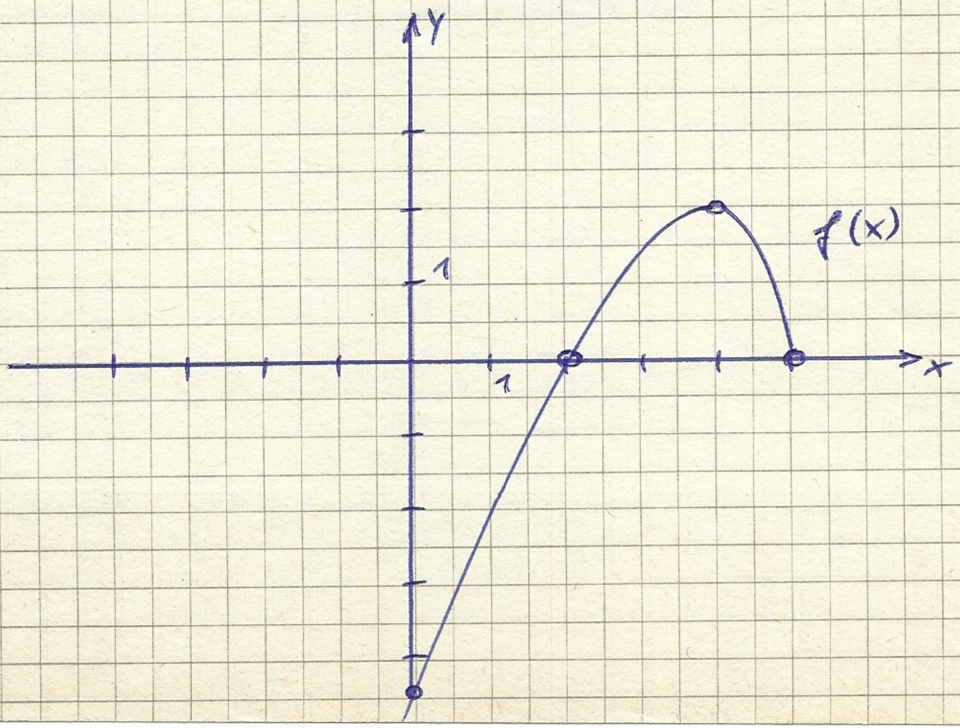
⑥ $x \to -\infty$:

$$\lim_{x\to-\infty} (x-2)\sqrt{5-x} = \lim_{x\to-\infty}(x-2)\cdot\lim_{x\to-\infty}\sqrt{5-x}$$

$$= -\infty \cdot +\infty$$

$$\underline{\underline{= -\infty}}$$

⑦



$$f(x) = 2x \cdot \sqrt{9-x^2}$$

1) ① $D = \{x,\ x \in R,\ |x| \leq 3\}$

$P_y(0|0)$

Punktsymmetrie:

$$f(x) = -f(-x)$$

$$2x\sqrt{9-x^2} = -\left(2\cdot(-x)\cdot\sqrt{9-(-x)^2}\right)$$

$$-\text{"}- \quad = 2x\sqrt{9-x^2} \quad \text{wahr}$$

---

2) Nullstellen:

$$0 = 2x \cdot \sqrt{9-x^2}$$

$$x_{0_1} = 0 \qquad x_{0_2} = 3\ ;\ x_{0_3} = -3$$

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3) Unstetigkeiten: keine

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4) lokale Extrempunkte:

$$f'(x) = 2\cdot\sqrt{9-x^2} + \cancel{2}x \cdot \frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2}}$$

$$= \frac{2(9-x^2) - 2x^2}{\sqrt{9-x^2}}$$

$$= \frac{18-4x^2}{\sqrt{9-x^2}}$$

Sün. B: $0 = 18 - 4x^2 \Rightarrow x_{E_1} = \sqrt{4{,}5}\ ;\ x_{E_2} = -\sqrt{4{,}5}$

$$f''(x) = \frac{-8x\sqrt{9-x^2} - (18-4x^2)\cdot\frac{-x}{\sqrt{9-x^2}}}{(9-x^2)}$$

$$= \frac{\frac{-8x(9-x^2) + (18-4x^2)x}{\sqrt{9-x^2}}}{9-x^2}$$

$$= \frac{4x^3 - 54x}{\sqrt{(9-x^2)^3}}$$

zu 4:

S. ü. h. B. $f''(\sqrt{4,5}) = \dfrac{4\cdot(\sqrt{4,5})^3 - 54\cdot\sqrt{4,5}}{\sqrt{(9-(\sqrt{4,5})^2)^3}}$

$= \dfrac{18\cdot\sqrt{4,5} - 54\cdot\sqrt{4,5}}{\sqrt{(9-4,5)^3}}$

$= \dfrac{-36\cdot\sqrt{4,5}}{\sqrt{(4,5)^3}} = \underline{\underline{-8}} < 0$

$\Rightarrow$ Max. $\Rightarrow$ $\underline{\underline{P_H(2,12 \mid 9)}}$

$f''(\sqrt{4,5}) = 8 > 0$ (aufgrund Symm.)

$\Rightarrow$ Min $\Rightarrow$ $\underline{\underline{P_T(-2,12 \mid -9)}}$

---

5) Wendepunkte:

$f''(x) = \dfrac{4x^3 - 54x}{\sqrt{(9-x^2)^3}}$

S. ü. u. B: $0 = 4x^3 - 54x$

$0 = x(4x^2 - 54)$

$\underline{\underline{x_{w_1} = 0}}$ $x_{w_{2,3}} = \pm\sqrt{13,5}$

$\hookrightarrow$ entfallen wg. DB!

S. ü. h B

Monotonie: $]\![-2 \mid 0)$: $f''(x) = \dfrac{x(4x^2-54)}{\sqrt{(9-x^2)^3}} = \dfrac{(-)\cdot(-)}{(+)} > 0$

$\Rightarrow$ monot. wachsend

$]\![0 \mid 2]$: $f''(x) = \dfrac{(+)\cdot(-)}{(+)} < 0 \Rightarrow$

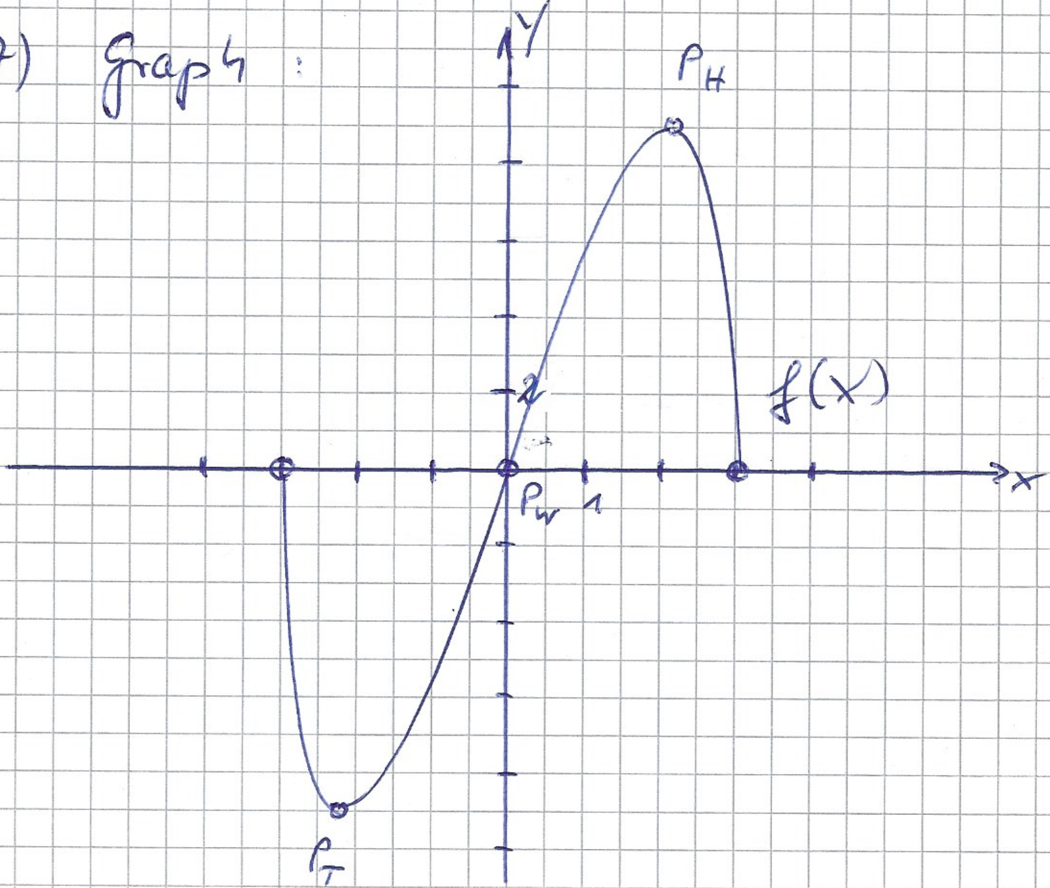
$\Rightarrow$ monot. fallend

$\Rightarrow$ Monotoniewechsel! $\Rightarrow$ f' hat hier Exp.

$\Rightarrow$ f hat hier $\underline{\text{WP}}$!

6) Verhalten $x \to \pm\infty$:
entfällt!

7) Graph:



HA: $f(x) = \frac{x^2+5x+22}{x-2}$ (1)

1. $DB = \{x; x \in \mathbb{R}; x \neq 2\}$

Symm: $f(x) = f(-x)$

$$\frac{x^2+5x+22}{x-2} = \frac{(-x)^2+5(-x)+22}{-x-2}$$

$$\neq \frac{x^2-5x+22}{-x-2}$$ kein Achsensymm.

$f(x) = -f(-x)$

$$\frac{x^2+5x+22}{x-2} = -\frac{(-x)^2+5(-x)+22}{-x-2}$$

$$\neq \frac{x^2-5x+22}{x+2}$$ keine Punktsymm.

$P_y(0|-11)$

---

2. Nst: $0 = \frac{x^2+5x+22}{x-2} \Rightarrow 0 = x^2+5x+22$

$x_{1,2} = -\frac{5}{2} \pm \sqrt{\underbrace{\frac{25}{4} - 22}_{D<0}}$!

keine Nst.

---

3. Unstetigkeiten: Vermutung $x = 2$

$\lim\limits_{x \to 2} \frac{x^2+5x+22}{x-2} = \frac{36}{0} \Rightarrow$ Polstelle $x_p = 2$

---

4. lok. E: $f'(x) = \frac{x^2-4x-32}{(x-2)^2}$

notw. B: $0 = x^2-4x-32 = (x+4)(x-8)$

$\Rightarrow x_{E_1} = -4 \; ; \; x_{E_2} = 8$

hinr. B: $f''(x) = \frac{72}{(x-2)^3}$

$f''(-4) = \frac{72}{(-6)^3} = -\frac{1}{3} < 0 \Rightarrow$ Max; $P_H(-4|-3)$

$f''(8) = \frac{72}{6^3} = \frac{1}{3} > 0 \Rightarrow$ Min; $P_T(8|21)$

---

5. WP: $f''(x) = \frac{72}{(x-2)^3}$; notw. B: $0 \neq 72$

$\Rightarrow$ keine WP

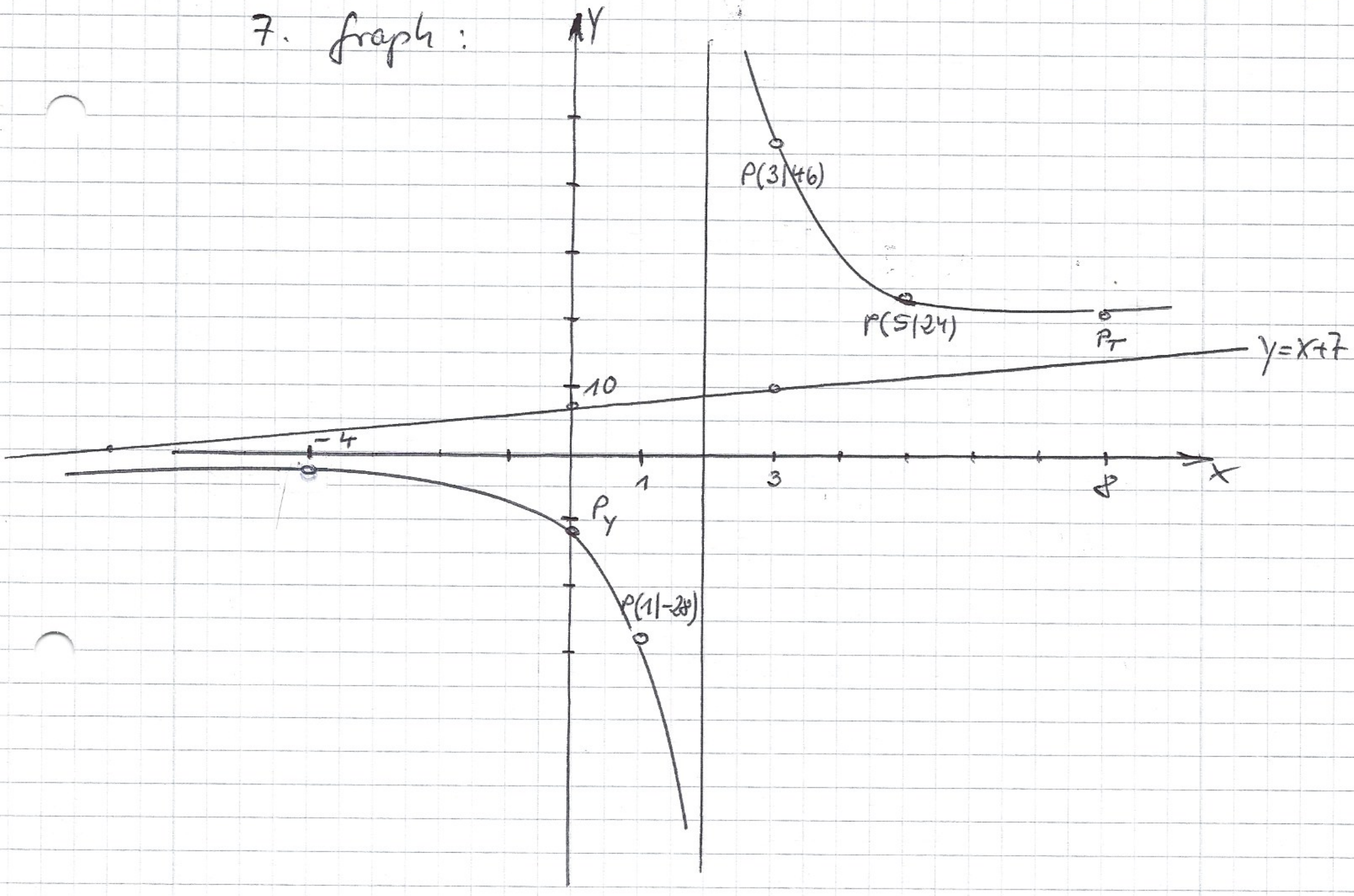
6. $x \to \pm\infty$: $\lim\limits_{x \to \pm\infty} \frac{x^2+5x+22}{x-2}$

$$= \lim_{x \to \pm\infty} \frac{\cancel{x}^{2}(1+\frac{5}{x}+\frac{22}{x^2})}{\cancel{x}(1-\frac{2}{x})} = \underline{\underline{\pm\infty}}$$

$$(x^2+5x+22):(x-2) = x+7+\frac{36}{x-2}$$
$$\frac{-(x^2-2x)}{\quad 7x+22}$$
$$\frac{-(7x-14)}{R\ 36}$$

schräge Asy: $\underline{\underline{y = x+7}}$

7. Graph:



$$f(x) = \frac{x-2}{x^2+5x+22}$$

1. $\mathbb{D}B = \{x ; x \in \mathbb{R}\}$; Keine Symm. ; $P_y(0|-\frac{1}{11})$

2. Nst: $0 = x-2 \curvearrowright \underline{\underline{x_0 = 2}}$

3. Unstetikeiten: Keine

4. Extrema:

$$f'(x) = \frac{-x^2+4x+32}{(x^2+5x+22)^2}$$

$\underline{nB:}$ $0 = -x^2+4x+32 \quad | \cdot(-1)$

$0 = x^2-4x-32$

$0 = (x-8)(x+4) \curvearrowright \underline{\underline{x_{E_1} = 8 ; x_{E_2} = -4}}$

$$f''(x) = \frac{2x^3-12x^2-192x-232}{(x^2+5x+22)^3}$$

$\underline{hB:}$

$f''(x_{E_1}) = f''(8) = -0{,}00076 < 0 \curvearrowright$ Max $P_H(8|0{,}048)$

$f''(-4) = 0{,}037 > 0 \curvearrowright$ Min $P_T(-4|-0{,}333)$

---

5. Wendepunkte:

$$f''(x) = \frac{2x^3-12x^2-192x-232}{(x^2+5x+22)^3}$$

$\underline{nB:}$ $0 = 2x^3-12x^2-192x-232$ nur mittels CP!

$x_{W_1} = -6{,}306$; $x_{W_2} = -1{,}347$; $x_{W_3} = 13{,}653$

$$f'''(x) = \frac{-6x^4+48x^3+1152x^2+2784x-744}{(x^2+5x+22)^4}$$

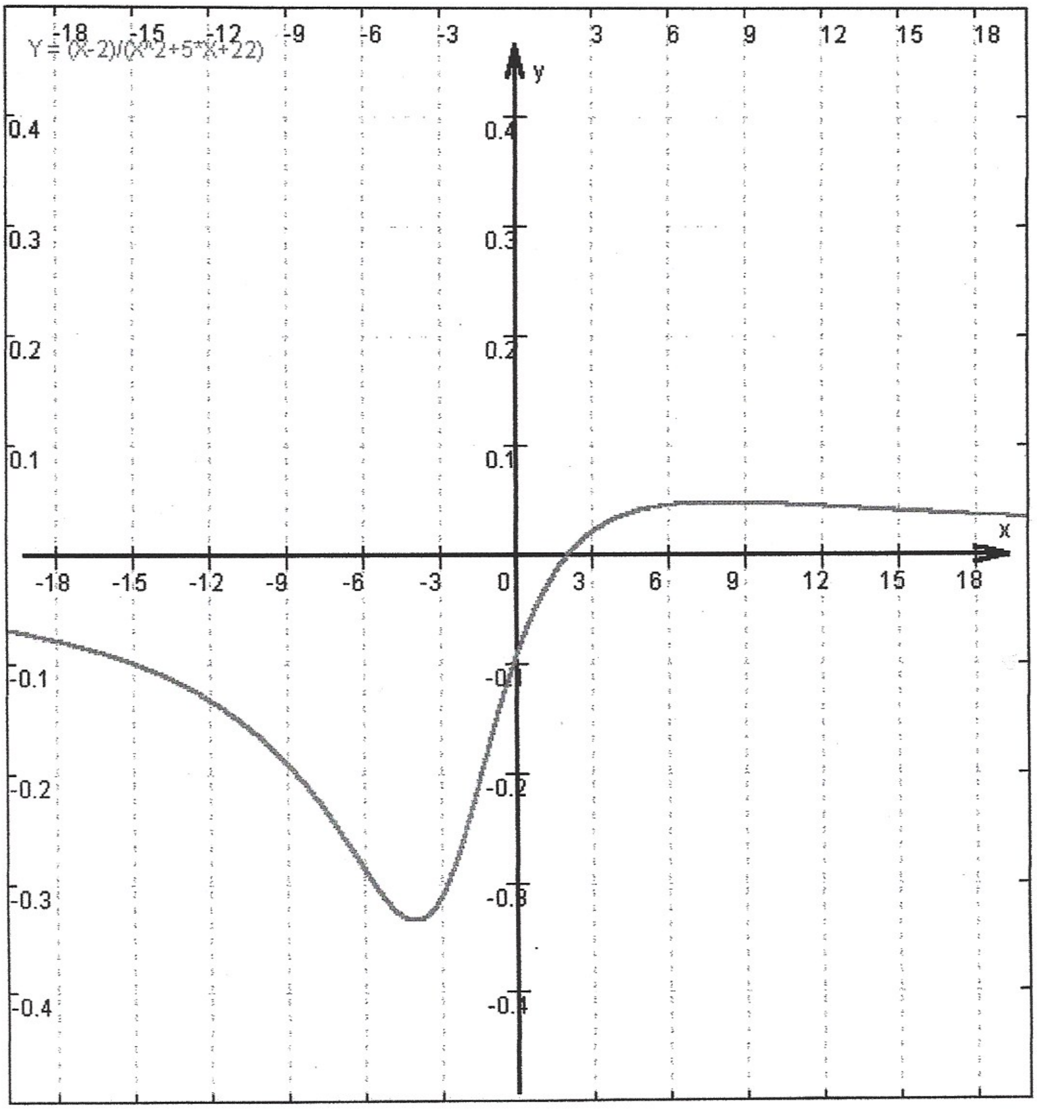
$\underline{h.B:}$

$f'''(-6{,}306) = 0{,}0072 > 0$ ReLiKu P_{W_1}

$f'''(-1{,}347) = -0{,}03 < 0$ LiReKu P_{W_2} → Folie

$f'''(13{,}653) = 0{,}000028 > 0$ ReLiKu P_{W_3}

6. $x \to \pm\infty$: $\lim\limits_{x \to \pm\infty} f(x) = 0 \curvearrowright$ Asy: $\underline{y = 0}$



Y = (X-2)/(X^2+5*X+22)
y
x
0.4
0.3
0.2
0.1
-0.1
-0.2
-0.3
-0.4
-18
-15
-12
-9
-6
-3
0
3
6
9
12
15
18

1

$f(x) = e^x \cdot \sin x$

1 $DB = \{x \mid x \in \mathbb{R}\}$, keine Symm., $P_y(0/0)$

Punktsymmetrie:

$f(x) = -f(-x)$

$e^x \sin x = -(e^{-x} \cdot \sin(-x))$

$= -\frac{1}{e^x} \cdot (-\sin(x))$

$e^x \sin x \neq \frac{1}{e^x} \cdot \sin x$

Achsensymmetrie:

$f(x) = f(-x)$

$e^x \sin x = e^{-x} \cdot \sin(-x)$

$= \frac{1}{e^x} \cdot (-\sin(x))$

$e^x \sin x \neq -\frac{1}{e^x} \sin x$

2 Nst: $0 = e^x \cdot \sin x$

$\neq 0$ $\quad$ $x_0 = \pi k$, $k \in \mathbb{Z}$

3 Unstetigkeiten: Keine

4 Extrema:

$f'(x) = e^x \cdot \sin x + e^x \cdot \cos x$

$f'(x) = e^x \cdot (\sin x + \cos x)$

n. B.: $0 = e^x (\sin x + \cos x)$

$\neq 0$

$0 = \sin x + \cos x \quad | - \cos x$

$-\cos x = \sin x \quad | : \cos x \neq 0!$

$-1 = \tan x$

$\Delta \quad x_E = -\frac{\pi}{4} + \pi k$

$f''(x) = e^x \cdot (\sin x + \cos x) + e^x \cdot (\cos x - \sin x)$

$f''(x) = e^x \cdot (2 \cos x)$

h. B.: $f''(-\frac{\pi}{4} + \pi k) = e^{-\frac{\pi}{4} + \pi k} \cdot 2 \cdot \cos(-\frac{\pi}{4} + \pi k)$

$= \underbrace{e^{-\frac{\pi}{4} + \pi k}}_{(+)} \cdot \underbrace{2}_{(+)} \cdot \underbrace{(\pm \frac{1}{2}\sqrt{2})}_{(\pm)} \gtrless 0$

Min / Max ✓

k gerade / k ungerade

②

$\curvearrowright$ $P_T\left(-\frac{\pi}{4}+\pi k \,/\, e^{-\frac{\pi}{4}+\pi k}\cdot \sin\left(-\frac{\pi}{4}+\pi k\right)\right)$ K gerade

$P_H\left(-\frac{\pi}{4}+\pi k \,/\, e^{-\frac{\pi}{4}+\pi k}\cdot \sin\left(-\frac{\pi}{4}+\pi k\right)\right)$ K ungerade

$\curvearrowright \ldots P_T\left(-\frac{\pi}{4} \,/\, -0{,}322\right)$ $K=0$ $(x_E \approx -0{,}78)$

$P_H\left(\frac{3}{4}\pi \,/\, 7{,}46\right)$ $K=1$ $(x_E \approx 2{,}36)$

$P_T\left(\frac{7}{4}\pi \,/\, -172{,}64\right) \ldots K=2$ $(x_E \approx 5{,}5)$

---

⑤ Wendepunkte:

$f''(x) = 2e^x \cdot \cos x$

n.B: $0 = \underbrace{2e^x}_{\neq 0} \cdot \cos x$

$\underline{\underline{x_W = \frac{\pi}{2} + \pi K}}$

$f'''(x) = 2e^x \cdot \cos x + 2e^x \cdot (-\sin x)$

$f'''(x) = 2e^x \cdot (\cos x - \sin x)$

h.B.: $f'''\left(\frac{\pi}{2}+\pi k\right) = \underbrace{2e^{\frac{\pi}{2}+\pi k}}_{(+)} \cdot \left(\cos\left(\frac{\pi}{2}+\pi k\right) - \sin\left(\frac{\pi}{2}+\pi k\right)\right)$

$(+) \cdot (0 \pm 1) \gtrless 0$

$\curvearrowright$ <u>ReLi / LiRe - Kurven</u>

K ungerade / K gerade

$\curvearrowright \ldots P_W(-1{,}57 \,/\, -0{,}21)$

$P_W(1{,}57 \,/\, 4{,}81)$

$P_W(4{,}71 \,/\, -111{,}32) \ldots$

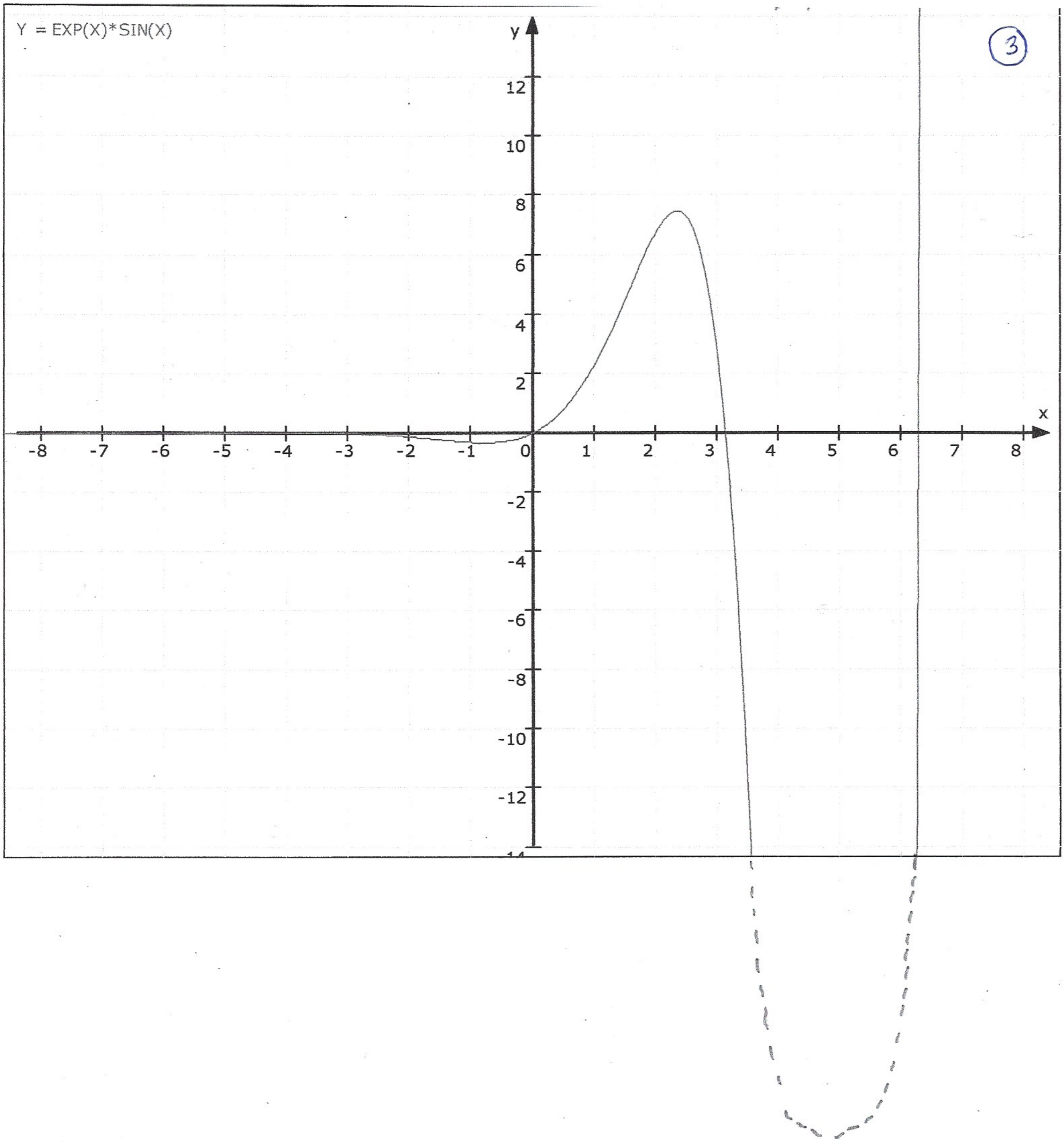
---

⑥ $x \to \pm\infty$: periodisch, wachsende Schwingung

$\lim\limits_{x \to \pm\infty} e^x \cdot \sin x = \lim\limits_{x \to \pm\infty} e^x \cdot \lim\limits_{x \to \pm\infty} \sin x$

$= \begin{bmatrix} +\infty \\ 0 \end{bmatrix} \cdot \pm 1$

$= \begin{bmatrix} \pm\infty \\ 0 \end{bmatrix}$ $\underline{\underline{y = 0}}$ Asymptote



Y = EXP(X)*SIN(X)
y
x
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
3